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E. Y. YU

**UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS**

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## Abstract

The problem of infinitely strong shock structure in a binary gas mixture is formulated using a bimodel approximation for each gas constituent. The bimodel consists of a delta function for the hypersonic flow upstream and a nonsingular function for the hot flow downstream. The Boltzmann equations are split into four equations. Under the local Maxwellian approximations for the nonsingular functions and an approximation on the force constants in the molecular interaction the problem is reduced to a numerical integration of three moment equations involving three non-dimensional parameters, namely, the mass ratio, the number density ratio, and the cut-off parameter ratio. The behavior of the singular points of the system of three moment equations at  $\pm$  infinity is analyzed, and a numerical integration scheme is proposed.



## INFINITELY STRONG SHOCK STRUCTURE IN A BINARY GAS MIXTURE

I. Introduction

The problem of shock structure in a gas mixture has in the recent few years received considerable attention. Theoretical works in the kinetic theory approach have been reported by Fujimoto,<sup>1</sup> Oguchi,<sup>2</sup> Oberai,<sup>3,4</sup> Beylich,<sup>5</sup> Harris and Bienkowski,<sup>6</sup> Sirovich and Goldman,<sup>7</sup> Abe and Oguchi.<sup>8</sup> Experimental measurements of shock profiles have been made by Center,<sup>9</sup> Schultz-Grunow,<sup>10</sup> and Beylich.<sup>11</sup> All these investigations concern weak shocks with a Mach number  $M$  around 2.0. (Beylich has performed his experiment also at  $M = 4.1$ .) No work, however, has yet been done on strong shock structure in a gas mixture.

In the present work, an attempt is made in solving for the shock profiles in a binary gas mixture under the limiting condition of infinite Mach number. The Mach number here is defined with respect to the upstream temperature which goes to zero in the limit relative to the downstream (hot side) temperature when the flow speed increases. As a result, the distribution function of the molecular particles in the far upstream becomes a delta function in the velocity space and acts like a molecular beam in the physical space. This family of particles then converts through collisions into those particles constituting the hot continuum flow downstream. This ansatz has been employed by Grad<sup>12</sup> and independently by Turcotte and Scholnick<sup>13</sup> in solving for the shock structure in a single monatomic gas. If the distribution function of the downstream hot particles is approximated by a local Maxwellian, as has been done by Grad, then there immediately results a symmetric density profile in the shock. If 13-moment approximation



is employed for the downstream particles, as attempted by Grad and the present author, then it is found that the shock profiles can only be obtained by numerically integrating a system of three moment equations. It is based on the result of the single gas that the present work is carried out since the foregoing ansatz can be simply extended to the case of gas mixture.

## II. Splitting of Solutions of Boltzmann Equations

We consider a steady, one-dimensional flow (in the x-direction) of a binary gas mixture, composing of gas constituents a and b with masses  $m_a$  and  $m_b$ . (Hereafter subscripts a and b refer to gas components a and b respectively.) At both far upstream ( $x = -\infty$ , denoted by superscript (0)) and far downstream ( $x = +\infty$ , denoted by superscript (1)) the gas constituents are in equilibrium with each other, as they are described by the Maxwellian distribution functions with common macroscopic velocities and temperatures  $u^{(0)}, T^{(0)}$  and  $u^{(1)}, T^{(1)}$  respectively. In particular, as  $T^{(0)} = 0$  at upstream, the Maxwellians there become delta functions, namely,

$$F_{ab}^{(0)} = n_{ab}^{(0)} \delta(\vec{\xi} - \vec{u}^{(0)})$$

where  $n_a^{(0)}, n_b^{(0)}$  are the number densities at  $x = -\infty$ , assumed to be given, and  $\vec{\xi}$  is the molecular velocity. The Boltzmann equations governing the change of a binary gas mixture from one equilibrium state to another are, in conventional notations,

$$\xi_x \frac{\partial F_a}{\partial x} = Q[F_a, F_a] + Q[F_a, F_b] \quad (1a)$$

$$\xi_x \frac{\partial F_b}{\partial x} = Q[F_b, F_b] + Q[F_b, F_a] \quad (1b)$$

where the expressions of the self and cross collision integrals can be found in standard references.<sup>14,15</sup> For reasons given in the Introduction, we now split the solutions into two parts

$$F_a = f_{\delta a} + f_a$$

$$F_b = f_{\delta b} + f_b$$

The singular parts

$$F_{\delta a} = n_{\delta a}(x) \delta(\vec{\xi} - \vec{u}^{(0)}) \quad (2)$$

come from upstream, and the nonsingular parts  $f_a, f_b$  dominate downstream.

It can be shown that the resulting four equations for the four functions are

$$\xi_x \frac{\partial f_{\delta a}}{\partial x} + (v_a + v_{b,a}) f_{\delta a} = Q[f_{\delta a}, f_{\delta a}] + Q[f_{\delta a}, f_{\delta b}] \quad (3)$$

$$\begin{aligned} \xi_x \frac{\partial f_a}{\partial x} + (v_{\delta a} + v_{\delta b,a}) f_a - 2Q_1[f_a, f_{\delta a}] - Q_1[f_a, f_{\delta b}] - Q_1[f_{\delta a}, f_b] \\ = Q[f_a, f_a] + Q[f_a, f_b] \end{aligned} \quad (4)$$

and two similar equations for  $f_{\delta b}$  and  $f_b$  by interchanging the subscripts  $a$  and  $b$  in Eqs. (3) and (4). Here,

$$v_a = \int f_{a1}(\xi_1) B_{aa} d\theta d\epsilon d\vec{\xi}_1 = 2\pi\beta_a n_a(x) \quad (5a)$$

$$v_{\delta a} = \int f_{\delta a1}(\xi_1) B_{aa} d\theta d\epsilon d\vec{\xi}_1 = 2\pi\beta_a n_{\delta a}(x) \quad (5b)$$

$$v_{b,a} = \int f_b B_{ab} d\theta d\epsilon d\vec{\xi}_b = 2\pi\beta_{ab} n_b(x) \quad (5c)$$

$$v_{\delta b,a} = \int f_{\delta b} B_{ab} d\theta d\epsilon d\vec{\xi}_b = 2\pi\beta_{ab} n_{\delta b}(x) \quad (5d)$$

in which the collision cross sections have been integrated under the assumption of a cut-off Maxwellian force interaction, namely,

$$\beta_a = \int B_{aa}(\theta) d\theta = A_{c.o.} (2\chi_{aa}/m_a)^{\frac{1}{2}} \quad (6a)$$

$$\beta_{ab} = \beta_{ba} = \int B_{ab}(\theta) d\theta = A_{c.o.} (m\chi_{ab}/m_a m_b)^{\frac{1}{2}} \quad (6b)$$

where  $A_{c.o.}$  is a constant (its magnitude depends on the cut-off),  $\chi_{aa}$ ,  $\chi_{ab}$  are force constants between a-a and a-b molecules, and  $m = m_a + m_b$ . Also,

$$2Q_1[f_a, f_{\delta a}] = \int [f'_a(\xi'_1) f'_{\delta a1}(\xi'_1) + f'_{a1}(\xi'_1) f'_{\delta a}(\xi'_1)] B_{aa} d\theta d\epsilon d\vec{\xi}_1 \quad (7a)$$

$$Q_1[f_a, f_{\delta b}] = \int [f'_a(\xi'_a) f'_{\delta b}(\xi'_b)] B_{ab} d\theta d\epsilon d\vec{\xi}_b \quad (7b)$$

$$Q_1[f_{\delta a}, f_b] = \int [f'_{\delta a}(\xi'_a) f'_b(\xi'_b)] B_{ab} d\theta d\epsilon d\vec{\xi}_b \quad (7c)$$

By integrating Eq. (3) and its counterpart equation on  $f_{\delta b}$  in the velocity space, we obtain



$$u^{(0)} \frac{dn_{\delta a}}{dx} + 2\pi[\beta_a n_a + \beta_{ab} n_b] n_{\delta a} = 0 \quad (8a)$$

$$u^{(0)} \frac{dn_{\delta b}}{dx} + 2\pi[\beta_b n_b + \beta_{ab} n_a] n_{\delta b} = 0 \quad (8b)$$

so that

$$n_{\delta a} = n_a^{(0)} \exp\left\{-\frac{1}{u^{(0)}} \int_{-\infty}^x 2\pi[\beta_a n_a(x') + \beta_{ab} n_b(x')] dx'\right\} \quad (9a)$$

$$n_{\delta b} = n_b^{(0)} \exp\left\{-\frac{1}{u^{(0)}} \int_{-\infty}^x 2\pi[\beta_b n_b(x') + \beta_{ab} n_a(x')] dx'\right\} \quad (9b)$$

This yields the solutions for  $f_{\delta a}$  and  $f_{\delta b}$  in (2) in terms of the zeroth moments,  $n_a$  and  $n_b$ , of  $f_a$  and  $f_b$ . Upon substituting  $n_{\delta a}$ ,  $n_{\delta b}$ ,  $f_{\delta a}$  and  $f_{\delta b}$  into Eq. (4) and its counterpart equation on  $f_b$  we obtain two self-contained equations for  $f_a$  and  $f_b$ . These two equations appear to be much more complex than the original Boltzmann equations (1a) and (1b), but they have the advantage of containing no singularities as the delta functions have been smoothed out through integrations so that they can be solved by some approximation methods. The boundary conditions of  $f_a$  and  $f_b$  are

$$f_a^{(-\infty)} = 0 \quad \text{as} \quad f_{\delta a}^{(-\infty)} = F_a^{(0)} \quad (10a)$$

$$f_a^{(+\infty)} = f_{ma}^{(1)} = \frac{n_a^{(1)}}{(2\pi R_a T^{(1)})^{3/2}} \exp\left\{-\frac{1}{2R_a T^{(1)}} (\vec{\xi} - \vec{u}^{(1)})^2\right\} \quad (10b)$$

(Similarly for  $f_b^{(+\infty)}$ ) as  $f_{\delta a}^{(+\infty)}$  and  $f_{\delta b}^{(+\infty)}$  are zero.

### III. Local Maxwellian Approximation

We shall now solve Eq. (4) and its counterpart equation on  $f_b$  by assuming that both  $f_a$  and  $f_b$  are local Maxwellian. We thus multiply these two equations successively by  $1, \xi_x, \xi^2$  and integrate them over the velocity space. The moments of the  $Q_1$  collision integrals can all be easily evaluated due to the presence of the delta functions. The moments of  $Q[f_a, f_b]$  are given as (see Kolodner,<sup>16</sup> Goldman and Sirovich, I,<sup>17</sup>)

$$I_{ab}^1 = \int m_a \xi_{ax} Q[f_a, f_b] d\vec{\xi}_a = -A_{ab} \rho_a \rho_b (u_a - u_b) \quad (11a)$$

$$I_{ab}^2 = \int m_a \xi_a^2 Q[f_a, f_b] d\vec{\xi}_a = -A_{ab} \rho_a \rho_b [2u_a (u_a - u_b) + \frac{6k}{m} (T_a - T_b) - \frac{2m_b}{m} (u_a - u_b)^2] \quad (11b)$$

and two similar expressions for the moments of  $Q[f_b, f_a]$ , denoted by  $I_{ba}^1 (= -I_{ab}^1)$  and  $I_{ba}^2 (= -I_{ab}^2)$ . The various quantities in (11a) and (11b) are defined by

$$\rho_a = m_a n_a = \int m_a f_a d\vec{\xi}_a \quad (12a)$$

$$u_a = \frac{1}{\rho_a} \int m_a \xi_{ax} f_a d\vec{\xi}_a \quad (12b)$$

$$n_a k T_a = p_a = \int \frac{1}{3} m_a (\vec{\xi}_a - \vec{u}_a)^2 f_a d\vec{\xi}_a \quad (12c)$$

where  $k$  is the Boltzmann constant, and

$$A_{ab} = 2\pi (A_1 / A_{c.o.}) (\beta_{ab} / m) \quad (12d)$$

where  $A_1 = 0.422$ . The resulting six moment equations are

$$\frac{d}{dx}(\rho_a u_a) + \frac{d}{dx}(\rho_{\delta a} u^{(0)}) = 0 \quad (13a)$$

$$\frac{d}{dx}(\rho_a u_a^2 + p_a) + \frac{d}{dx}(\rho_{\delta a} u^{(0)2}) = I_{ab}^1 \quad (13b)$$

$$\frac{d}{dx}(\rho_a u_a^3 + 5p_a u_a) + \frac{d}{dx}(\rho_{\delta a} u^{(0)3}) = I_{ab}^2 \quad (13c)$$

$$\frac{d}{dx}(\rho_b u_b) + \frac{d}{dx}(\rho_{\delta b} u^{(0)}) = 0 \quad (14a)$$

$$\frac{d}{dx}(\rho_b u_b^2 + p_b) + \frac{d}{dx}(\rho_{\delta b} u^{(0)2}) = I_{ba}^1 \quad (14b)$$

$$\frac{d}{dx}(\rho_b u_b^3 + 5p_b u_b) + \frac{d}{dx}(\rho_{\delta b} u^{(0)3}) = I_{ba}^2 \quad (14c)$$

for the six unknowns,  $\rho_a$ ,  $u_a$ ,  $p_a$ ,  $\rho_b$ ,  $u_b$ , and  $p_b$ . Eqs. (13a) and (13b) can readily be integrated with the integration constants evaluated at  $x = -\infty$ , yielding the mass conservation equations

$$\rho_a u_a + \rho_{\delta a} u^{(0)} = \rho_a^{(0)} u^{(0)} \quad (15a)$$

$$\rho_b u_b + \rho_{\delta b} u^{(0)} = \rho_b^{(0)} u^{(0)} \quad (15b)$$

At this point we introduce the macroscopic quantities for the composite gas as we will see later it is more convenient to work with the equations of the composite gas. Define (see Goldman and Sirovich<sup>17</sup>)



$$\rho_{\delta} = \rho_{\delta a} + \rho_{\delta b}$$

$$n = n_a + n_b = \int (f_a + f_b) d\vec{\xi}$$

$$\rho = \rho_a + \rho_b = \int (m_a f_a + m_b f_b) d\vec{\xi}$$

$$u = \frac{1}{\rho} (\rho_a u_a + \rho_b u_b) = \frac{1}{\rho} \int [m_a \xi_x f_a + m_b \xi_x f_b] d\vec{\xi}$$

$$p = nkT = \frac{1}{3} \int [m_a f_a + m_b f_b] (\vec{\xi} - \vec{u})^2 d\vec{\xi} = p_a + p_b + \frac{\rho_a \rho_b}{\rho} [u_a^2 + u_b^2 - 2u_a u_b]$$

$$P_{xx} = \int (m_a f_a + m_b f_b) (\xi_x - u) (\xi_x - u) d\vec{\xi}$$

$$Q_x = \frac{1}{2} \int (m_a f_a + m_b f_b) (\xi_x - u) (\vec{\xi} - \vec{u})^2 d\vec{\xi}$$

In the present local Maxwellian approximation for  $f_a$  and  $f_b$ , the stress  $P_{xx}$  and the heat flow  $Q_x$  are zero so that there exist two relations among  $\rho_a$ ,  $u_a$ ,  $p_a$  and  $\rho_b$ ,  $u_b$ ,  $p_a$ . The conservation equations for  $\rho$ ,  $u$ , and  $p$  of the composite gas can be shown to be

$$\frac{d}{dx} (\rho u + \rho_{\delta} u^{(0)}) = 0 \quad (16a)$$

$$\frac{d}{dx} (\rho u^2 + p + \rho_{\delta} u^{(0)2}) = 0 \quad (16b)$$

$$\frac{d}{dx} (\rho u^3 + 5pu + \rho_{\delta} u^{(0)3}) = 0 \quad (16c)$$

or, in integrated form,

$$\rho u + \rho_{\delta} u^{(0)} = \rho^{(0)} u^{(0)} \quad (17a)$$

$$\rho u^2 + p + \rho_{\delta} u^{(0)2} = \rho^{(0)} u^{(0)2} \quad (17b)$$

$$\rho u^3 + 5pu + \rho_{\delta} u^{(0)3} = \rho^{(0)} u^{(0)3} \quad (17c)$$

The solution<sup>12</sup> of (17a,b,c) is

$$\rho = 4(\rho^{(0)} - \rho_{\delta}) \quad (18a)$$

$$p = \frac{3}{16} u^{(0)2} \rho \quad (18b)$$

whereas  $u$  and  $RT = p/\rho$  are constant throughout, i.e.,

$$u = \frac{1}{4} u^{(0)} \quad (18c)$$

$$RT = \frac{3}{16} u^{(0)2} \quad (18d)$$

We will now consider Eqs. (13a,b,c) together with Eqs. (16a,b,c) instead of Eqs. (14a,b,c) as  $\rho_b$ ,  $u_b$ ,  $p_b$  of the gas constituent  $b$  can be replaced by  $\rho$ ,  $u$ ,  $p$  of the composite gas. As a result, the dependent variables are reduced from six to four, namely,  $\rho_a$ ,  $u_a$ ,  $p_a$ , and  $\rho$ , by virtue of the solution (18a,b,c).

Let us now introduce the following nondimensional variables:

$$\tilde{\rho}_i = \rho_i / \rho^{(0)}, \quad \tilde{n}_i = n_i / n^{(0)}$$

$$\tilde{u}_i = u_i / u^{(0)}, \quad \tilde{p}_i = p_i / \rho^{(0)} u^{(0)^2}$$

$$\tilde{\rho}_{\delta i} = \rho_{\delta i} / \rho^{(0)}, \quad i = a, b$$

$$\tilde{\rho} = \rho / \rho^{(0)},$$

$$\tilde{u} = u / u^{(0)} = \frac{1}{4}$$

$$\tilde{p} = p / \rho^{(0)} u^{(0)^2} = \frac{3}{16} \tilde{\rho} = \tilde{p}_a + \tilde{p}_b + \frac{\tilde{\rho}_a \tilde{\rho}_b}{\tilde{\rho}} [\tilde{u}_a - \tilde{u}_b]^2$$

where  $\rho^{(0)} = \rho_a^{(0)} + \rho_b^{(0)} = m_a n_a^{(0)} + m_b n_b^{(0)}$ , and  $n^{(0)} = n_a^{(0)} + n_b^{(0)}$ .

We also nondimensionalize  $x$  with respect to a length scale  $\lambda$  which has the meaning of mean free path defined on the hot side, i.e.,

$$\tilde{x} = x / \lambda, \quad \lambda = \frac{m u^{(0)}}{2\pi \beta_{ab} \rho^{(1)}}$$

where  $\rho^{(1)} = 4\rho^{(0)}$  as indicated by (17a). The ratios of the integrated cross sections are given as

$$\tilde{\beta}_a = \frac{\beta_a}{\beta_{ab}} = \left( \frac{2m_b}{m} \frac{\chi_{aa}}{\chi_{ab}} \right)^{\frac{1}{2}}$$

$$\tilde{\beta}_b = \frac{\beta_b}{\beta_{ab}} = \left( \frac{2m_a}{m} \frac{\chi_{bb}}{\chi_{ab}} \right)^{\frac{1}{2}}$$

$$A^* = \frac{A_{ab}}{2\pi \beta_{ab} / m} = \frac{A_l}{A_{c.o.}}$$



It is seen that there are altogether five parameters in the problem, namely,

the mass ratio,  $m_b/m_a = M$

the number density ratio,  $n_b^{(0)}/n_a^{(0)} = N$

the force constant ratios,  $\chi_{aa}/\chi_{ab}, \chi_{bb}/\chi_{ab}$

and the cut off parameter ratio,  $0 < A^* = A_1/A_{c.o.} < 1$

Here the first four parameters are given for given gas constituents a and b, but the value of the last parameter  $A^*$  has to be assigned arbitrarily.

Let us also denote

$$\psi_a = \frac{\rho_a^{(0)}}{\rho^{(0)}} = \frac{1}{1+MN}, \quad \psi_b = \frac{\rho_b^{(0)}}{\rho^{(0)}} = 1 - \psi_a$$

$$M_a = \frac{m_a}{m} = \frac{1}{1+M}$$

$$M_b = \frac{m_b}{m} = \frac{M}{1+M}$$

In nondimensional form, the resulting four equations for  $\tilde{\rho}$ ,  $\tilde{\rho}_a$ ,  $\tilde{u}_a$ , and  $\tilde{p}_a$  are

$$\frac{d\tilde{p}}{d\tilde{x}} = 4h(\tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a) + \left[ \frac{\tilde{\beta}}{M_b} \tilde{\rho} + \left( \frac{1}{M_a} - \frac{\tilde{\beta}}{M_b} \right) \tilde{\rho}_a \right] (\psi_b + \tilde{\rho}_a \tilde{u}_a - \frac{\tilde{p}}{4}) \quad (19a)$$

$$\frac{d}{d\tilde{x}} (\tilde{\rho}_a \tilde{u}_a) = h(\tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a) \quad (19b)$$

$$\frac{d}{d\tilde{x}} (\tilde{\rho}_a \tilde{u}_a^2 + \tilde{p}_a) = h(\tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a) - \frac{A^*}{4} \tilde{\rho} \tilde{\rho}_a (\tilde{u}_a - \frac{1}{4}) \quad (19c)$$

$$\begin{aligned} \frac{d}{dx}(\tilde{\rho}_a \tilde{u}_a^3 + 5\tilde{p}_a \tilde{u}_a) &= h(\tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a) - \frac{A^*}{4} [2\tilde{\rho} \tilde{p}_a \tilde{u}_a (\tilde{u}_a - \frac{1}{4}) + \\ &6[M_a \tilde{p}_a (\tilde{\rho} - \tilde{\rho}_a) - \frac{3}{16} M_b \tilde{\rho} \tilde{p}_a + M_b \tilde{\rho}_a \tilde{p}_a] - 2M_b \tilde{\rho} \tilde{p}_a (3\tilde{\rho}_a + \tilde{\rho}) (\tilde{u}_a - \frac{1}{4})^2 (\tilde{\rho} - \tilde{\rho}_a)^{-1}] \end{aligned} \quad (19d)$$

where

$$h(\tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a) (= - \frac{d}{dx} \tilde{\rho} \delta_a) = \frac{1}{4} \left[ \left( \frac{\tilde{\rho}}{M_a} - \frac{1}{M_b} \right) \tilde{\rho}_a + \frac{1}{M_b} \tilde{\rho} \right] (\psi_a - \tilde{\rho}_a \tilde{u}_a)$$

The boundary conditions for this system of equations are: at  $x = -\infty$

$$\tilde{\rho}^{(0)} = \tilde{\rho}_a^{(0)} = \tilde{u}_a^{(0)} = \tilde{p}_a^{(0)} = 0, \text{ and at } x = +\infty, \tilde{\rho}^{(1)} = 4, \tilde{\rho}_a^{(1)} = 4\psi_a, \tilde{u}_a^{(1)} = \frac{1}{4}, \text{ and}$$

$$\tilde{p}_a^{(1)} = \frac{3\psi_a M_b}{4\psi_a M_b + 4\psi_b M_a} = \frac{3}{4(1+N)}. \text{ To obtain the solution for } \tilde{\rho}, \tilde{\rho}_a, \tilde{u}_a, \text{ and } \tilde{p}_a$$

it is necessary to numerically integrate the system of four equations between the above two singular points. Such a task proves to be formidable, since the direction lines from the singular points in the 4-dimensional phase space are difficult to analyze. In what follows, we shall make certain assumptions in order to further simplify the problem for a possible solution.

#### IV. Approximate Solutions

We now assume that the force constant in the interaction between molecular particles is proportional to the mass of the particles, i.e.,

$$\chi_{aa} : \chi_{bb} : \chi_{ab} = m_a : m_b : \frac{2m_a m_b}{m_a + m_b} \quad (20)$$

Consequently, the integrated collision cross sections become equal to one another,

$$\beta_a = \beta_b = \beta_{ab} = \beta_0 \quad (20a)$$

so that  $\tilde{\beta}_a = 1$  and  $\tilde{\beta}_b = 1$ . Thus we are now left with only three parameters,  $M$ ,  $N$ , and  $A^*$ .

Under the foregoing assumption, it follows from (9a), (15a), and (17a) that

$$\tilde{\rho}_\delta = \exp\left[-\frac{1}{u(0)} \int_{-\infty}^x 2\pi\beta_0 n(x') dx'\right] \quad (21a)$$

$$\tilde{\rho}_{\delta a} = \psi_a \tilde{\rho}_\delta = \psi_a \left(1 - \frac{1}{4} \tilde{\rho}\right) \quad (21b)$$

and

$$\tilde{\rho}_a \tilde{u}_a = \frac{1}{4} \psi_a \tilde{\rho} \quad (21c)$$

where the last relation can now be used to eliminate  $\tilde{u}_a$  from (19c) and (19d). Finally, we obtain a system of three equations for  $\tilde{\rho}$ ,  $\tilde{\rho}_a$ , and  $\tilde{p}_a$ , reduced to the following form:

$$\frac{d\tilde{\rho}}{dx} = g_1(\tilde{\rho}, \tilde{\rho}_a) \quad (20a)$$

$$\begin{aligned} \Delta_2 \frac{d\tilde{\rho}_a}{dx} = g_2(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) = & \left[ \frac{5}{4} \psi_a \tilde{\rho} \tilde{\rho}_a - \frac{7}{16} \psi_a \tilde{\rho}^2 - \tilde{\rho}_a^2 + 5\tilde{p}_a \tilde{\rho}_a \right] \tilde{\rho}_a g_1(\tilde{\rho}, \tilde{\rho}_a) \\ & - \frac{3}{16} A^* \tilde{\rho}^2 \tilde{\rho}_a^2 (\psi_a \tilde{\rho} - \tilde{\rho}_a) + \frac{A^*}{\psi_a} \tilde{\rho}_a g(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) \end{aligned} \quad (20b)$$

$$\begin{aligned} \Delta_3 \frac{d\tilde{p}_a}{dx} = g_3(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) = & \left[ \frac{1}{32} \psi_a \tilde{\rho}^3 \tilde{\rho}_a^2 + \frac{5}{4} \psi_a \tilde{\rho} \tilde{\rho}_a^2 - \frac{1}{256} \psi_a \tilde{\rho}^4 - \frac{5}{16} \psi_a \tilde{\rho} \tilde{\rho}_a^2 \right. \\ & \left. - \frac{1}{16} \psi_a \tilde{\rho} \tilde{\rho}_a^2 \right] g_1(\tilde{\rho}, \tilde{\rho}_a) - \frac{5}{16} A^* \tilde{\rho} \tilde{\rho}_a^2 \tilde{p}_a (\psi_a \tilde{\rho} - \tilde{\rho}_a) + \frac{A^*}{16} \psi_a \tilde{\rho} g \end{aligned} \quad (20c)$$



where

$$g_1(\tilde{\rho}, \tilde{\rho}_a) = \left[ \left( \frac{1}{M_a} - \frac{1}{M_b} \right) \tilde{\rho}_a + \frac{1}{M_b} \tilde{\rho} \right] \left( 1 - \frac{1}{4} \tilde{\rho} \right)$$

$$g_2(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) = 6M_a \tilde{\rho}_a^2 \tilde{p}_a (\tilde{\rho} - \tilde{\rho}_a) - 6M_b \tilde{\rho}_a^2 \left[ \frac{3}{16} \tilde{\rho} \tilde{\rho}_a - \tilde{p}_a \tilde{\rho}_a - \frac{1}{16} \frac{\tilde{\rho} (\psi_a \tilde{\rho} - \tilde{\rho}_a)}{(\tilde{\rho} - \tilde{\rho}_a)} \right] \\ - \frac{1}{8M_b} \frac{\tilde{\rho}_a \tilde{\rho}^2 (\psi_a \tilde{\rho} - \tilde{\rho}_a)^2}{(\tilde{\rho} - \tilde{\rho}_a)}$$

$$\Delta_2 = \tilde{\rho} \left[ -\frac{3}{16} \psi_a \tilde{\rho}^2 + 5 \tilde{p}_a \tilde{\rho}_a \right]$$

$$\Delta_3 = \tilde{\rho}_a \Delta_2 / \tilde{\rho}$$

To effect a numerical integration of the above three equations we first combine them into two equations by letting  $\tilde{\rho}$  be the independent variable in the phase space:

$$\frac{d\tilde{\rho}_a}{d\tilde{\rho}} = \frac{g_2}{\Delta_2 g_1} = G_1(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) \quad (21a)$$

$$\frac{d\tilde{p}_a}{d\tilde{\rho}} = \frac{g_3}{\Delta_3 g_1} = G_2(\tilde{\rho}, \tilde{\rho}_a, \tilde{p}_a) \quad (21b)$$

The singular points are

$$\text{at } x = -\infty: \quad \tilde{\rho}^{(0)} = 0, \quad \tilde{\rho}_a^{(0)} = 0, \quad \tilde{p}_a^{(0)} = 0$$

$$\text{at } x = +\infty: \quad \tilde{\rho}^{(1)} = 4, \quad \tilde{\rho}_a^{(1)} = \frac{4}{1+MN}, \quad \tilde{p}_a^{(1)} = \frac{3}{4(1+N)}$$

Whether or not there exists an integrated curve connecting the two singular

points in the phase space depends on the nature of the singularity of these two points, which shall now be analyzed.

(A) Singularity at  $x = -\infty$ . Near the singular point at  $x = -\infty$  we treat  $\tilde{\rho}$ ,  $\tilde{\rho}_a$ ,  $\tilde{p}_a$  to be of the same order of smallness. By neglecting terms of the second order in  $\tilde{\rho}$ ,  $\tilde{\rho}_a$  on the right hand side of Eq. (20a) and terms of the fifth order in  $\tilde{\rho}$ ,  $\tilde{\rho}_a$ ,  $\tilde{p}_a$  on the right hand side of Eqs. (20b) and (20c), we obtain

$$\frac{d\tilde{\rho}}{dx} = \left(\frac{1}{M_a} - \frac{1}{M_b}\right)\tilde{\rho}_a + \frac{1}{M_b}\tilde{\rho} \quad (22a)$$

$$\tilde{\rho} \left( -\frac{3}{16}\psi_a \tilde{\rho}^2 + 5\tilde{p}_a \tilde{\rho}_a \right) \frac{d\tilde{\rho}_a}{dx} = \left[ \frac{5}{4}\psi_a \tilde{\rho} \tilde{\rho}_a - \frac{7}{16}\psi_a \tilde{\rho}^2 - \tilde{\rho}_a^2 + 5\tilde{p}_a \tilde{\rho}_a \right] \tilde{\rho}_a \quad (23b)$$

$$\left[ \left(\frac{1}{M_a} - \frac{1}{M_b}\right)\tilde{\rho}_a + \frac{1}{M_b}\tilde{\rho} \right]$$

$$\tilde{\rho}_a \left( -\frac{3}{16}\psi_a \tilde{\rho}^2 + 5\tilde{p}_a \tilde{\rho}_a \right) \frac{d\tilde{p}_a}{dx} = \left[ \frac{1}{32}\psi_a \tilde{\rho}^2 \tilde{\rho}_a + \frac{5}{4}\tilde{\rho} \tilde{p}_a - \frac{1}{256}\psi_a \tilde{\rho}^3 - \frac{5}{16}\psi_a \tilde{\rho} \tilde{\rho}_a \tilde{p}_a \right. \\ \left. - \frac{1}{16}\psi_a \tilde{\rho} \tilde{\rho}_a^2 \right] \psi_a \left[ \left(\frac{1}{M_a} - \frac{1}{M_b}\right)\tilde{\rho}_a + \frac{1}{M_b}\tilde{\rho} \right] \quad (23c)$$

If we let  $\tilde{\rho}_a = \theta_1 \tilde{\rho}$  and  $\tilde{p}_a = \theta_2 \tilde{\rho}$  as  $\tilde{\rho}$  approaches zero, we obtain two algebraic equations on  $\theta_1$  and  $\theta_2$ ,

$$\theta_1^2 - \frac{5}{4}\psi_a \theta_1 + \frac{1}{4}\psi_a^2 = 0 \quad (24a)$$

$$5\theta_1^2 \theta_2^2 + \left( \frac{1}{8}\psi_a^2 - \frac{5}{4}\psi_a \theta_1 \right) \theta_1 \theta_2 + \left( \frac{1}{256}\psi_a^4 + \frac{1}{16}\psi_a^2 \theta_1^2 - \frac{1}{32}\psi_a^3 \theta_1 \right) = 0 \quad (24b)$$

The solution of (24a) and (24b) is

$$(i) \quad \theta_1 = \psi_a, \quad \theta_2 = \begin{cases} (3/16)\psi_a \\ (3/80)\psi_a \end{cases} \quad (25)$$

$$(ii) \quad \theta_1 = \frac{1}{4}\psi_a, \quad \theta_2 = \begin{cases} (3/20)\psi_a \\ 0 \quad (\text{disregarded}) \end{cases}$$

These three sets of values of  $\theta_1$  and  $\theta_2$  represent the directions of three field lines coming off from the singular point. Along any one of the field lines,  $\tilde{\rho}_a$ ,  $\tilde{p}_a$  and  $\tilde{\rho} \sim \exp(\mu_1 \tilde{x})$  approaches zero as  $x \rightarrow -\infty$ , where  $\mu_1 = \frac{1}{M_a}\theta_1 + (1-\theta_1)\frac{1}{M_b}$  is always positive for either value of  $\theta_1 (<1)$ . This high order singularity at  $x = -\infty$  has the behavior equivalent to a saddle point in the conventional two-dimensional phase space. Apparently this saddle point has three asymptotes in the three-dimensional phase space.

(B) Singularity at  $x = +\infty$ . Let us now linearize Eqs. (22a, b, c) about the singular point at  $x = +\infty$ , i.e.,  $\tilde{\rho} = \tilde{\rho}^{(1)} + \tilde{\rho}'$ ,  $\tilde{\rho}_a = \tilde{\rho}_a^{(1)} + \tilde{\rho}_a'$ , and  $\tilde{p}_a = \tilde{p}_a^{(1)} + \tilde{p}_a'$ . The resulting linearized equations are

$$\frac{d\rho'}{dx'} = -\mu_2 \rho' \quad (26a)$$

$$\frac{d\rho_a'}{dx'} = b_1 \rho_a' + c_1 p_a' + d_1 \rho' \quad (26b)$$

$$\frac{dp_a'}{dx'} = b_2 \rho_a' + c_2 p_a' + d_2 \rho' \quad (26c)$$



where  $x' = (A^*/4)\tilde{x}$ ,  $\mu_2 = \frac{4(1+M)(1+N)}{A^*(1+MN)} (>0)$

$$b_1 = \frac{16(-5MN+M+N-5)}{(1+M)(5MN-N+4)}$$

$$c_1 = \frac{512 M (1+N)^2}{(1+M)(1+MN)(5MN-N+4)}$$

$$d_1 = \frac{16(1+N)}{(5MN-N+4)} \left\{ \mu_2 \left[ \frac{1}{1+MN} + \frac{5}{1+N} \right] - \frac{1+7M}{(1+MN)(1+M)} + \frac{6}{(1+M)(1+N)} \right\}$$

$$b_2 = \frac{5M^2N - 19MN + 5M + 18N - 1}{(1+M)(5MN-N+4)}$$

$$c_2 = \frac{32M(1+N)^2}{(1+M)(1+MN)(5MN-N+4)}$$

$$d_2 = \frac{1}{3(5MN-N+4)} \left\{ 8(1+N) + 9\mu_2(5-N) + \frac{1}{1+MN} \left[ 18\frac{1-M}{1+M} - (17+2N+15MN) \right] \right\}$$

Note that these coefficients have all been expressed in terms of the three parameters,  $M$ ,  $N$ , and  $A^*$ . Let us introduce the transformation

$$\sigma = \xi \rho_a' + \eta p_a' + \zeta \rho' \quad (27)$$

such that

$$\frac{d\sigma}{dx'} = s\sigma \quad (28)$$

Substitution of (27) and (26a,b,c) into (28) yields

$$\begin{aligned} (b_1-s)\xi + b_2\eta &= 0 \\ c_1\xi + (c_2-s)\eta &= 0 \\ d_1\xi + d_2\eta - (\mu_2+s) &= 0 \end{aligned} \quad (29)$$

The solution of the characteristic equation of the above algebraic equations is the following three eigenvalues

$$s_1 = \frac{1}{2} \{ (b_1 + c_2) \pm [(b_1 + c_2)^2 - 4(b_1 c_2 + c_1 b_2)]^{\frac{1}{2}} \}$$

$$s_3 = -\mu \quad (< 0)$$
(30)

It can be easily verified that  $(b_1 c_2 + c_1 b_2)$  is always negative. Hence, one of the first two eigenvalues is always negative, which is designated by  $s_2$ . The eigenvectors corresponding to  $s_1$  and  $s_2$  are

$$\xi_i : \eta_i : \zeta_i = -b_2(\mu_2 + s_i) : (b_1 - s_i)(\mu_2 + s_i) : [(b_1 - s_i)d_2 - d_1 b_2], \quad i=1,2 \quad (31)$$

and the eigenvector corresponding to  $s_3$  is  $0:0:1$  or simply the  $\rho'$ -axis.

These are the three eigendirections in the  $\rho_a' p_a' \rho'$  phase space, with two ( $s_2$  and  $s_3$ ) accessible to and one ( $s_1$ ) inaccessible to the singular point as  $x \rightarrow +\infty$ . Finally, let us express  $\rho_a'$ ,  $p_a'$ ,  $\rho'$  in terms of  $\sigma_i = R_i \exp(s_i x')$ ,  $i = 1, 2, 3$ , i.e.,

$$\begin{pmatrix} \rho_a' \\ p_a' \\ \rho' \end{pmatrix} = \begin{pmatrix} \xi_1 & \eta_1 & \zeta_1 \\ \xi_2 & \eta_2 & \zeta_2 \\ 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} \quad (32)$$

If  $\sigma_1$  is taken to be zero, then

$$\rho_a' = \frac{1}{(\xi_1 \eta_1 - \xi_2 \eta_1)} [-\eta_1 \sigma_2 + (\eta_1 \zeta_2 - \eta_2 \zeta_1) \rho'] \quad (33a)$$

$$p_a' = \frac{1}{(\xi_1 \eta_2 - \xi_2 \eta_1)} [\xi_1 \sigma_2 - (\xi_1 \zeta_2 - \xi_2 \zeta_1) \rho'] \quad (33b)$$

where  $\sigma_2$  and  $\rho'$  may be arbitrarily chosen as will be seen later.

#### V. A Numerical Integration Scheme

To find the shock profile in a binary gas mixture, it is necessary to numerically integrate Eqs. (21a) and (21b), where  $\tilde{\rho}$  is an independent variable. In view of the nature of the singularities as discussed in the last section, it seems necessary to start integrations from the singular points at both ends. At  $x = -\infty$  there are three direction lines emerging from the singular point. It is not known a priori which direction line connects to the singular point at  $x = +\infty$ . Therefore, one has to try every one of them in the numerical work. The starting point of the integration is taken to be

$$\tilde{\rho}_a^* = \theta_1 \tilde{\rho}, \quad \tilde{p}_a^* = \theta_1 \tilde{p}$$

where  $\theta_1, \theta_2$  are given in (25), and  $\tilde{\rho}(>0)$  is the arbitrarily chosen initial value. At  $x = +\infty$ , the initial direction line may be taken along any one on the plane passing through the eigenvectors labelled by  $\sigma_2$  and  $\sigma_3$ . Then the starting point is

$$\tilde{\rho}_a^* = \tilde{\rho}_a^{(1)} + \rho_a', \quad \tilde{p}_a^* = \tilde{p}_a^{(1)} + p_a'$$

where  $\rho_a'$  and  $p_a'$  are given by (33a) and (33b) in which  $\rho'$  is negative. By



varying  $\sigma_2$  and the step size  $\rho'$  at the plus end one could probably get the integrated curve intercept one of the three curves from the minus end with a continuous slope at the interception point. Such a continuous integrated curve,  $\tilde{p}_a(\tilde{\rho})$  and  $\tilde{p}_a(\tilde{\rho})$ , are a solution of Eqs. (21a) and (21b). Finally, this curve in the phase space is transformed back to the physical space,  $\tilde{p}(x)$ ,  $\tilde{p}_a(x)$ ,  $\tilde{p}_a(x)$ , through the use of Eq. (20a).

The numerical values of the mass ratio  $M = m_b/m_a$ , and the number density ratio  $N = n_b^{(0)}/n_a^{(0)}$ , have to be chosen in compliance with a realistic model of gas mixture as, for example,

(a) Ar - He mixture

$$M = \frac{m_{\text{Ar}}}{m_{\text{He}}} = 10, \quad 0.01 \leq N.$$

(b)  $N_2$  -  $H_2$  mixture

$$M = \frac{m_{N_2}}{m_{H_2}} = 7, \quad 0.05 \leq N.$$

The magnitude of the third parameter, i.e., the cut-off parameter  $A^*$ , ranges between 0 and 1, i.e.,  $0 < A^* < 1$ . The effect of  $A^*$  on shock structure can only be investigated numerically.

Eqs. (20a,b,c) or Eqs. (21a,b) will have to be programmed on a digital computer for numerical integrations. Such a work will be undertaken shortly.

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Houston, Texas 77001

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Charlottesville, Virginia 22903

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Ohio University  
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Professor James A. Cadzow  
Department of Electrical Engineering  
State University of New York at Buffalo  
Buffalo, New York 14214

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Navy Underwater Sound Laboratory  
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## ERRATUM

Mr Jerome Fox, Research Coordinator  
Polytechnic Institute of Brooklyn  
55 Johnson St (should be 333 Jay St)  
Brooklyn, N.Y. 11201

## DELETE

Mr Morton M. Pavane, Chief  
AFSC Scientific & Tech. Liaison Office  
26 Federal Plaza, Suite 1313  
New York, New York 10007

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13. ABSTRACT  The problem of infinitely strong shock structure in a binary gas mixture is formulated using a bimodel approximation for each gas constituent. The bimodel consists of a delta function for the hypersonic flow upstream and a nonsingular function for the hot flow downstream. The Boltzmann equations are split into four equations. Under the local Maxwellian approximations for the nonsingular functions and an approximations on the force constants in the molecular interaction the problem is reduced to a numerical integration of three moment equation the problem is reduced to a numerical integration of three moment equations involving three nondimensional parameters, namely, the mass ratio, the number density ratio, and the cut-off parameter ratio. The behavior of the singular points of the system of three moment equations at $\pm$ infinity is analyzed, and a numerical integration is proposed.			



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## KEY WORDS

## LINK A

## LINK B

## LINK C

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